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THE STABILITY OF SHIPS

EXPLAINED SIMPLY AND CALCULATED BY A NEW GRAPHIC METHOD.

BY

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I HEREBY CERTIFY that I visited, on the 17th inst., MESSRS. M. HOLZAPFEL & CO.'S Manufactory for preparing RAHTJEN'S ANTI-FOULING COMPOSITION FOR SHIPS' BOTTOMS, and saw there, in large quantities and of GOOD QUALITY, all the necessary raw materials for making this Composition, and the excellent machinery for thoroughly mixing it, and that I also took a sample of the Composition then being manufactured, and have analyzed it. From these examinations I am SATISFIED THAT THIS COMPOSITION IS BEING MANUFACTURED BY Messrs. M. HOLZAPFEL & Co. in accordance with the instructions given in the SPECIFICATION OF RAHTJEN'S PATENT.

JOHN PATTINSON, F.I.C., F.C.S.

S.S. *Tongariro*, 4,163 tons, of the NEW ZEALAND SHIPPING COMPANY, London,

Previous to steaming 12,410 miles in 40 days 9 hours and 27 minutes, and 11,725 miles in 38 days 3 hours and 5 minutes, the whole circuit of the world, out and home, in 78 days 12 hours 32 minutes, the *Tongariro*, AS WELL AS THE OTHER STEAMERS OF THIS COMPANY, WERE SUPPLIED WITH COMPOSITION BY US.

All Casks, Drums, &c., containing Composition manufactured by us, bear our Name and Mark, a guarantee of Full Strength and Finest Quality.

We are exceedingly obliged to the gentlemen who have kindly favoured us with a statement of their experience of our INTERNATIONAL Composition. The best testimonials, however, we continue to receive from our competitors, when they decline to practically test the respective merits of the Compositions, by refusing to coat one side of a ship against our Composition on the other side. We let this fact speak for itself.

SUNDERLAND, *November 3rd*, 1883.

We beg to inform you that we tried your INTERNATIONAL COMPOSITION on our S.S. *Ahdeck*, against ....'s Composition. When we slipped her in Hull last month, our Mr. Butchart examined her bottom, and found that ....'s Composition was covered with grass and small shells all over, while your Composition was still quite clean and fresh.

We were highly pleased with the result of your Composition, and ordered it to be applied to the whole ship.

Yours faithfully, (Signed) WALKER AND BUTCHART.

Telegram, London, 13th September, 1882.

S.S. *William Burkitt*. Your Paint's superiority being acknowledged far beyond doubt by everyone present, we are re-coating Port side.

Trial of INTERNATIONAL on Port side against .... Composition—First Trial.

Telegram, London, March 5th, 1883.

S.S. *William Burkitt*. Dock now dry. Our side again far better than the other. We were highly complimented for the efficiency of the INTERNATIONAL COMPOSITION. We are re-coating same side.

Trial of INTERNATIONAL on Port side against .... Composition—Second Trial.

Telegram, Antwerp, May 23rd, 1883.

.... Paint Starboard-side has come off altogether after one Odessa voyage, our side being quite clean and in good order. She is being painted with INTERNATIONAL on Starboard side only, *Port side* not requiring anything.

INTERNATIONAL on both sides.

NEWPORT, MON., *November 9th*, 1883.

Undersigned having carefully examined the bottom of S.S. *Hartlepools*, now lying in Messrs. Mordey and Carney's Dry Dock in Newport, finds that the Starboard side, down to bilge, was covered with grass and a good many barnacles, and the bottom was covered with slime and some barnacles. The paint was to a great extent washed off and the iron pitted badly.

The Port side had a green slime on down to bilge plate, and a few traces of small shells in the run: the bottom was still perfectly clean and fresh, had a very good cover of paint on, and was perfectly free from rust.

(Signed) GEORGE DARTNELL, MARINE SURVEYOR.

The Port side was painted with INTERNATIONAL Composition.
The Starboard side with ....'s Composition.

RECENT TRIALS, ONE SIDE ONLY INTERNATIONAL.

S.S. *Minerva*..................	INTERNATIONAL against	..................................'s Composition.
S.S. *Triumph*	INTERNATIONAL against	....'s Composition.
S.S. *Embleton*	INTERNATIONAL against	..................................'s Composition.
S.S. *Ashburne*	INTERNATIONAL against	....'s Composition.
S.S. *Harbinger*..............	INTERNATIONAL against	..................................'s Composition.

THE STABILITY OF SHIPS

ERRATA.

Page 6, line 28.—*For* " the falling of B while B is supported,"
read " the falling of G while B is supported."

Page 6, line 4 from bottom.—*For* " as at G, then the weight at G tends,"
read " as at G_1, then the weight at G_1 tends."

Page 7, line 18 from bottom.—*For* " when G is above M as at G_1 and G_2,"
read " when G is below M as at G_1 and G_2."

Page 8. line 6.—*For* " B_{20} Z_{20}," *read* " G_{20} Z_{20}."

Page 8, line 7.—*For* " B_{30} Z_{30}," *read* " G_{30} Z_{30}."

Page 12, line 7.—*For* " Fig. 37, Plate IV.," *read* " Fig. 37, Plate II."

Page 15, line 6 from bottom.—*For* " the centre of that co-ordinate,"
read " the centre of that ordinate."

Page 16, line 7 from bottom.—*For* " $\frac{X_1 7}{2m} \frac{X_1}{7 \times 2m}$ " $=$ *read* " $\frac{X_1 7}{2m} = \frac{X_1}{7 \times 2m}$."

Page 27, line 6 from bottom.—*For* " half the scale of Fig. 34,"
read " half the scale of Fig. 35."

In Fig. 35.—The vertical line corresponding to a displacement of 1,500 tons should be marked " C C."

LAMBERT & CO., LIMITED, PRINTERS, GREY STREET, NEWCASTLE.

1884.

PREFACE.

THE new and original part of this work, "THE GRAPHIC CALCULATION OF THE DATA, DEPENDING ON FORM, REQUIRED FOR DETERMINING THE STABILITY OF SHIPS," was submitted to the Institution of Naval Architects, in a paper read by the Author at their annual meeting this year, 1884.

Until then, all calculations for the stability of ships were made by determining the moments of the wedges of immersion and emersion, a process involving considerable labour, and giving results applicable to only one displacement. At this year's meeting of the Institution of Naval Architects, three papers on this subject were read, and in all of these the above method, which had hitherto been universal, was abandoned, and new methods were given, by which centres of buoyancy corresponding to any angle of heel and to any displacement can be determined directly. Of these new methods, the one invented by the Author and given here is much the simplest. It involves no calculations but what can be made by the T square and scale. Its accuracy depends, like every other method, on the skill of the draughtsman, with this advantage—which is common to all graphic methods—that gross errors are detected at sight, and smaller errors are checked by the fairness of the curves worked out.

J. C. S.

THE STATICAL STABILITY OF SHIPS.

The great loss of life and of valuable property, resulting from accidents to ships, gives, to all matters relating to their safety, an importance which it would be difficult to over-estimate.

The stability of a ship, or the force with which it tends to keep upright, is evidently of vital importance in its behaviour and safety.

This truth is now being generally recognised, and from recent decisions in Courts of Enquiry, it would appear that all who are responsible for the designing, building, loading, and navigating of vessels are expected to understand and know their stability.

At present, few, except Scientific Naval Architects, do understand this subject, and consequently, the calculations of stability, which would be of great practical value, if understood by those responsible for the working of ships, are little more than mathematical curiosities.

The laws relating to the static stability of floating bodies are exceedingly simple, and can be understood by anyone who understands the action of a lever.

The application of these laws to ships is equally simple in principle. The only difficulty in the matter is due to their shapes, which are such that each water line and frame section is in general different from every other, and each of these alters in shape and position for every angle to which the ship may be inclined.

The object of the following is, first, to explain, so as to be intelligible to those who have no technical knowledge of Naval Architecture, the principles involved in calculations of stability. And, secondly, to give a new graphic method, which is simpler, easier, and more comprehensive than that at present in use, for making the calculations of the forms of ships.

PART I.

THE PRINCIPLES INVOLVED IN CALCULATIONS OF STABILITY.

A SHIP floating at rest in still water will usually float upright. To heel her over to any angle from the vertical requires a certain force. This force, called the righting moment, because it tends to bring the ship back to the vertical if heeled over by the action of external forces, such as wind or waves, is a measure of the stability of a ship that is of the resistance which it would offer to being overturned.

To determine the righting moment at every angle of heel, for a ship under any conditions of loading, is to determine the statical stability of that ship.

It is called the statical stability because the calculations are made on the supposition that the ship is at rest in still water.

A ship at sea is subject to many other complex forces in addition to the righting moment; but these forces are variable and practically incalculable, whilst the righting moment depends only on the ship and her cargo, and can be determined for every ship.

The upright position of a ship is always taken as the normal position about which the stability is calculated. But it is by no means uncommon to have ships which do not tend to float upright, thus many ships cannot stand up without a certain amount of ballast, others may have a list from improper stowage of cargo, or from Cargo shifting, or from fixed weights, such as the engines not being central.

In all such cases, many of which are of the greatest importance and interest, the stability is calculated about the upright position, proper allowance being made for all the conditions affecting each case.

DATA REQUIRED FOR CALCULATIONS OF STABILITY.

The righting moment of any ship at any angle of heel depends on the following factors:—

1st. The total weight of ship and cargo.

2nd. The position of the centre of gravity of that weight.

3rd. The angle of inclination.

4th. The centre of buoyancy corresponding to that angle.

WEIGHT OF SHIP.

The weight of a ship is here used to mean the total weight of the ship itself, and of all that it contains under any given conditions.

For any given draught a ship displaces a certain quantity of water, dependent on its form and dimensions. The weight of this displaced water is the same as that of the ship, and is given in a "displacement scale," which is made out for every ship.

CENTRE OF GRAVITY.

A ship, like every other heavy body, has a centre of gravity, that is a point where we may suppose the whole weight of the ship to be concentrated, and which must be supported in order that the body may be at rest.

The position of this point depends partly on the distribution of materials in the ship itself, and partly on the nature and stowage of the cargo. The nature and stowage of cargo vary, more or less, everytime the ship is loaded, making it very difficult to calculate the position of the centre of gravity; this, however, is unnecessary, as it can be found experimentally by a very simple process, to be afterwards explained.

CENTRE OF BUOYANCY.

If Fig. 1 represent a vessel floating in water to the level W L, it is evident that the weight of the ship must be balanced by the pressure of the water against its bottom and sides.

It is also evident that the ship takes the place of a mass of water W K L which formerly occupied the same space. It follows that the resultant of the pressure of the water at all points of the surface of the vessel is just equivalent to the weight of a mass of water which would occupy the space W K L.

But the weight of water filling the space W K L is equivalent to a single force concentrated at a point B, the centre of gravity of that space. The buoyancy of the water surrounding W K L must therefore be exactly equivalent to this same force acting through the same point.

The centre of buoyancy is therefore the centre of gravity of the displaced water.

As the two sides of a ship are alike, the centre of buoyancy will be in the centre line of ship when it is upright.

But when the ship is heeled over to an angle as in Fig. 2, it displaces more water on the low side and less water on the high side than when the vessel is upright. This shifts the centre of buoyancy to a new position, B, which will lie on that side of the centre line on which the immersion has been increased. To find the position of this point B corresponding to all angles of heel and to all draughts of water must always involve a long and tedious calculation; but as it depends only on the form of the ship, and not on the loading, it may be done once for all and tabulated for each ship.

CALCULATION OF STABILITY.

The factors enumerated above being known for any ship, the calculation of her stability is a process of the simplest possible description.

Thus if Fig. 3 represent a ship inclined out of the perpendicular, G being the centre of gravity of the ship, and B the centre of buoyancy at that inclination, we have the weight of the ship acting vertically downwards through G, and the buoyancy of the water acting vertically upwards through B, and we have to determine the effect of these two forces. Suppose that the ship, instead of being water borne, were supported by blocks directly below B, as in Fig. 3.

It is evident that the ship would not balance in this position, for the centre of gravity, not being supported, would fall lower, and the falling of B while B was supported would tend to turn the centre line of ship towards the vertical.

The same reasoning applies to the case when the ship is afloat at this inclination.

The buoyancy of the water acting through the point B cannot balance the force of gravity acting at the point G, and the centre of gravity not being directly supported will fall, tending to turn the ship into her vertical position, where the buoyancy and gravity acting in the same straight line (the centre line of ship) will balance.

Now, suppose that by taking the cargo out of a ship and placing it on deck the centre of gravity was raised to the position G, in Fig 4.

It is again evident that a supporting force through B could not balance the force of gravity at G, and the centre of gravity not being supported would fall lower, in this case, turning the centre line of ship further away from its upright position; and C would continue to fall until the increasing inclination of the ship shifted B exactly under G, or if that did not happen the ship would turn completely over.

If M, Fig. 5, is the point where the perpendicular through B cuts the centre line of ship, it is evident that if the centre of gravity is anywhere below M, as at G, then the weight at G tends to right the ship. But if the centre of gravity be above M, as at G_2, then the weight at G_2 tends to upset the ship.

If the centre of gravity were at M the forces would balance.

To express this technically we say that if G is below M the ship has a positive righting moment; if G is above M the ship has a negative righting moment; if B is at M the righting moment=O. We have now to measure these moments.

THE MEASUREMENT OF MOMENTS.

If W, Fig 6, is a weight suspended at the end of a lever supported at its fulcrum F, the weight tends to turn the lever about this fulcrum, and the force with which it tends to turn the lever is equal to the weight multiplied by its distance from the fulcrum. This product is the moment of the weight about F, it may be expressed in inch lbs. or foot tons, or in any other convenient units of weight and length.

Thus if, in Fig 6, $W=3$ lbs., $l=5$ inches, the moment about F will be $5\times 3=15$ inch lbs. If $W=3$ tons, $l=5$ feet, the moment would be 15 foot tons.

The righting moment for ships is generally expressed in foot tons; that is the weight of the ship in tons is multiplied by the length of leverage in feet.

If W, Fig. 7, is a weight at the end of a pendulum, its moment about F is its weight multiplied by l, the horizontal distance of W from F or in general terms.

If a weight is supported by a force applied at a distance from its centre of gravity, a moment will be produced equal to the weight multiplied by the horizontal distance between the centre of gravity and the point of support.

This moment always tends to move the centre of gravity of the weight to a position vertically below the point of support. This law is of course true for ships supported by the buoyancy of the water they float in, hence if B is the centre of buoyancy of a ship inclined to an angle as at Fig 8. B M the vertical line through B; G_1 is the centre of gravity of the ship, and G_1 Z_1 is the horizontal distance between the point G_1 and the line B M. Then the moment of the ship about B equals the weight of the ship multiplied by the length G_1 Z.

And this is the force that must be used to incline the ship to this angle, and conversely. If the ship is inclined to this angle by the action of an external force it tends to return to the vertical with a force equal to this moment.

The length of the arm G Z evidently varies as the distance of G from M. Thus if G_1, G_2, G_3, and G_4, are different positions of the centres of gravity corresponding to different stowage of cargo for the same draught of water.

We see that when G is above M as at G_1 and G_2, the corresponding leverages G_1 Z_1 and G_2 Z_2 increase as the distance from M to G increases, and that the righting moments are positive.

But if G is above M as at G_3 and G_4 the moments are negative tending to upset the ship, and that the leverage of these moments increase as the height of G above M is increased.

We may now restate the fundamental principle of stability, as follows:—

The weight of a ship is treated as a single force acting through its centre of gravity. The buoyancy of the water in which a ship floats is treated as a single force equal to the weight of the ship, and acting through the centre of buoyancy. If these two forces are in the same straight line, they will neutralise each other. If they are not in the same straight line they will not neutralize each other, but will produce a moment (equal to the weight of the ship multiplied by the horizontal distance between the lines of action of the gravity and the buoyancy). This moment will tend either to right or to upset the ship according to the relative positions of these centres.

Now, the weight of a ship and the position of its centre of gravity are not altered by inclining the ship to different angles. (It is assumed that no weight shifts when the ship is inclined). But the position of B, and consequently the leverage G Z, is different for different angles.

To determine the righting moment for any angle, it is therefore only necessary to find the length G Z corresponding to that angle, as this length when multiplied by the weight of the ship is the required moment.

The usual way of representing the stability of a ship, at all angles, is by means of a diagram called a curve of stability, and constructed as follows:—

The length of the arm G Z is found for each 10° of inclination; a straight line A B, Fig 10, is taken and divided into equal parts, each of which is supposed to represent 10° of heel.

Then G_{10} Z_{10} is drawn square to A B and equal to the length of G Z corresponding to 10° of heel.
„ B_{20} Z_{20} „ „ „ „ „ „ 20° „
„ B_{30} Z_{30} „ „ „ „ „ „ 30° „ &c.

The lengths of G Z being measured above A B where the righting moment is positive.

The lengths of G Z being measured below A B where the righting moment is negative.

A curve drawn through all the points Z_{10}, Z_{20}, Z_{30}, &c., is a complete index of the stability of the ship; for to find the righting moment at any angle it is only necessary to measure the height of the curve at that angle, and multiply it by the weight of the ship, noting that if the curve is above the line the moment is positive, if below, negative.

Before shewing how the positions of B and M may be practically determined for any ship, an examination of some of the properties of the meta centre will be advisable.

THE META CENTRE.

It has been shewn that if M, Fig. 8, is the point where the centre line of ship intersects the vertical through the centre of buoyancy, the righting moment depends on the relative positions of the points M and G. Now as the point M depends only on the position of the centre of buoyancy, and as the centre of buoyancy depends only on the form of ship and the angle of inclination, it follows that, for any given displacement and angle of heel, the position of M is a fixed point, dependent on the form of ship, but independent of the condition of loading. And for any given displacement the position of M depends only on the angle of heel.

When the angle of heel is indefinitely small the point M is called the meta centre, and for small angles of heel up to about 10° *the position of M is nearly constant.*

And if G, Fig 9, is the centre of gravity of the ship, then G M is called the meta centric height.

For very small angles of heel the meta centric height is all that is required to be known to determine the stability of the ship. Thus to find the arm of righting moment corresponding to the inclination shewn in Fig. 9, through G draw G Z square to M Z, then G Z is the required arm.

For although we may not know the exact position of B, we know it is somewhere in the vertical through M, that is in the line M B, and that the distance from the point G to the line M B is the arm of the righting moment. Now, it has been proved (see Fig 8), that where M is the intersection of the vertical through the centre of buoyancy with the centre line of ship, that G Z the arm of the righting moment varies directly as the distance from G to M; also that the righting moment is positive when G is below M and negative when G is above M, in which case M G, the meta centric height, is said to be negative.

Thus, the meta centric height (positive or negative) is a sufficient measure of the stability of a ship for small angles of inclination.

For large angles of inclination, the verticals through the centres of buoyancy do not (in general) intersect the centre line at the meta centre. And for such angles the meta centric method of estimating stability does not apply.

It then becomes necessary to find from the ship's drawings the actual position of M, corresponding to these large angles, because ships roll to much larger angles than those to which the meta centric method is applicable; and usually danger to the ship only commence when the angle of inclination is considerable.

Thus the meta centric height, though not a sufficient measure of the stability of a ship for large angles of inclination, is so for small angles, and determines what is called initial stability—that is, stability about the upright position.

The meta centric height thus determines if a ship is to be stiff, tender, or crank.

For if the centre of gravity is near the "meta centre," the arm of the righting moments will be small, and the ship easily inclined, that is, it will be tender.

As G M, the meta centric height increases, the arm of righting moment, and consequently the resistance of ship to being inclined, increases. That is, the ship becomes stiffer as the meta centric height increases. Again, if G is above M the ship will not be able to stand upright, and the higher G is above M the greater is the upsetting moment—that is, the ship becomes more crank as the negative meta centric height increases. As the stiffness of a ship increases directly with the meta centric height, it might be inferred that the greater this height the better; this, however, is not the case, for as the meta centric height increases the period of oscillation of the ship decreases, causing the ship to become uneasy in a sea-way. That meta centric height which gives the best results can only be found by experience of the behaviour of a ship at sea, but this belongs to a much larger subject than statical stability.

The meta centric height is also used in determining, by an inclining experiment, the actual position of the centre of gravity of a ship, as will be subsequently explained.

We may now summarise the foregoing remarks on the meta centre as follows:—

The meta centre is a point depending on the form of a ship.

The meta centric height is the height of the meta centre above the centre of gravity.

If the meta centre is below the centre of gravity, the ship is said to have negative meta centric height.

The meta centric height determines the initial stability of a ship.

If the meta centric height is great, the ship is stiff.

If the meta centric height is small, the ship is tender.

If the meta centric height is negative, the ship is crank.

We will now show how the meta centric height is to be found experimentally for any ship loaded in any way.

THE CENTRE OF GRAVITY OF SHIPS.

To find, by an inclining experiment, the position of the centre of gravity of a ship, the ship is trimmed, as nearly as possible, to the exact upright position; a known weight is then shifted to a given distance from the centre line, this inclines the ship to the same side. The angle to which the ship is inclined is observed. The shifting weight is then brought back to the centre; this should, if no other weights have shifted, bring the ship back to its original upright position.

The same weight is then moved to the opposite side and the corresponding angle of heel noted. Finally the weight is brought back to the centre, when the ship should also return to the upright.

Care must be taken that the only weights shifted are those with which the experiment are being made. Free water in bilges, tanks, boilers, or any other weights that might easily shift without being observed, would vitiate the results of the experiment.

Careful observation to find if the ship returns exactly to the upright position each time the shifting weight is brought to the centre, will usually detect if any other weights have shifted.

If great accuracy is desired the experiment is repeated with different shifting weights, with a view of detecting and correcting any error in experiment or observation.

If perfectly free from error a single experiment is sufficient to determine the meta centric height, and (the position of the meta centre being known) the position of the centre of gravity.

An example will best illustrate this: given a ship, whose total weight is 1,000 tons, which is inclined 10° by shifting a weight of 10 tons to a distance of 10 feet from the centre.

To find its meta centric height we first find the distance to which the centre of gravity of ship is shifted by shifting the weight. Thus, when the weights are central, and the ship is floating upright, the centre of gravity will be somewhere in the centre line of the ship; suppose it to be at G_1, Fig. 11.

Let 10 tons be now shifted to a distance of 10 feet from the centre, as at W_2, Fig. 11, the total weight no longer balances about the centre line, but forms a turning moment about it, equal to 10 tons $\times$ 10 feet $=$ 100 foot tons.

The centre of gravity corresponding to this new distribution of weight must be a point at such a distance from the centre line that, if the whole weight of ship were concentrated there, it would produce the same moment. Let G_2, Fig. 11, be this point, and G_1 G_2 the horizontal distance of G_2 from centre line, then G_1 $G_2 \times 1{,}000$ tons, must equal 100 foot tons; or, G_1 $G_2 = \frac{100}{1000} = \frac{1}{10}$ feet.

That is to say, shifting a weight of 10 tons to a distance of 10 feet from the centre, will shift the centre of gravity of a ship weighing 1,000 tons $\frac{1}{10}$th of a foot.

In the above we assume that G_1 and G_2 are at the same level or height above the keel, and this will be the case, for altering the thwartship position of a weight will not raise or lower the centre of gravity.

Having found G_1 G_2 the alteration in centre of gravity of ship, due to shifting the weight from centre to side, and knowing the corresponding angle to which the ship is inclined, (namely, 10°) the meta centric height, M G_1, Fig. 12, is found as follows:—

As the ship is in equilibrium at 10° of heel, the centre of gravity must be in the line of action of the buoyancy; that is, G_2 must be somewhere in the line B M, which is inclined at 10° to the centre line, and which passes through M, the meta centre. (See definition of meta centre.)

Making, therefore, G_1 G_2, Fig. 13, equal to $\frac{1}{10}$th foot on any scale; and drawing, through G_1, the line G_1 M vertically upwards; and drawing, through G_2, the line G_2 M, inclined at 10° to the vertical. The point M, where they intersect, will give G_1 M the height of meta centre = ·56 feet.

Lastly, if M, the meta centre corresponding to a displacement of 1,000 tons, is known, the actual position of G is found by measuring ·56 feet below M.

From the above example, it will be seen that the process of determining experimentally the centre of gravity of a ship, may be reduced to the following simple rules:—

1st. Find the angle to which a ship is inclined by shifting a known weight to a given distance from the centre.

2nd. The moment (in foot tons) produced by this shifted weight, divided by the weight of the ship in tons, gives the distance (in feet) that the centre of gravity of the whole ship is altered by moving the weight.

3rd. Draw a horizontal line G_1 G_2 equal to the length last found, through G_1 draw a vertical line. Through G_2 draw a line, inclined to the vertical, at the same angle to which the ship is inclined. The point M where these lines meet, gives the height of M above G, that is the meta centric height.

4th. If the position of meta centre is known, measure down a distance equal to G_1 M from the point M, which will give G, the position of the centre of gravity, when the weights are central and the ship upright.

Or to express the same process by symbols.

Let w = the shifting weight in tons; l = the distance to which it is shifted; w l = the resultant moment.

Let W = the weight of ship; g = the distance the C of G of ship is altered by shifting weight, then $g\,W = w\,l$ and $g = \dfrac{w\,l}{W}$

If H = meta centric height.

ϕ = angle to which the ship is inclined by the shifting of weight.

$$\frac{H}{g} = \text{cotangent}\ \phi$$

$$\text{Therefore } H = g \text{ cotangent } \phi = \frac{w\,l}{W} \text{cotangent } \phi$$

The process given above applies only to ships with positive meta centric heights.

Ships with negative meta centric heights do not float upright, and are only in equilibrium at an angle of heel which is necessarily greater than that to which the meta centric method applies.

If we know the centre of buoyancy corresponding to the angle of heel at which a top heavy ship rests, the centre of gravity may be determined at once. Thus, if B, Fig. 14, is the centre of buoyancy corresponding to the angle of heel at which a ship, with negative meta centric height, is in equilibrium, through B draw the vertical line. The point G, where this intersects the centre line of ship, will be the centre of gravity of ship; for when the ship is in equilibrium, G must be vertically over B, and the centre of gravity is in the centre line of ship.

The practical details of arranging and shifting the weights, and of measuring the angles of inclination, and of preventing any other weights from shifting—although they require great care and skill—are too obvious to need any comment, and the calculations required are of the simplest descripton; and even this small calculation can be saved by the use of the stability indicator invented by Mr. A. Taylor.

In this instrument the moveable weights consist of tanks containing a known weight of water, which may be emptied or filled as required. A gauge is fitted to shew, with great sensitiveness, the angle of inclination of the ship, and scales are adjusted to this gauge, which shew, by inspection, the meta centre heights corresponding to the angle of inclination.

If the meta centric height measured the stability of ships, this instrument would be a complete gauge of the stability of the ship. But as ships with the same meta centric height may have very different values for their stability, at large angles of inclination, it is not of itself a gauge of this stability, but it supplies, in a very simple and convenient form, data essential for determining the stability.

Let us now recapitulate the principles of the foregoing pages.

The stability of a ship is the force with which it resists the action of external forces which tend to overturn the ship.

It depends on 3 factors, viz.:—The weight of the ship.

The position of its centre of gravity.

The line of action through the centre of buoyancy.

Of these:—The centre of gravity may be found by a simple inclining experiment.

The weight of ship may be found from "displacement scale."

The line of action through the centre of buoyancy may be found from a diagram, similar to Fig. 35, which gives the height of the point M (referred to in Fig. 5), corresponding to any displacement, and to a series of angles of heel.

This diagram contains all the data, depending on the form of ship, required for calculating a ship's stability at any displacement. And a curve of stability can be constructed from this diagram, if the draft of water and meta centric height are known.

Thus suppose the ship, of which Fig. 35 is the diagram, to be loaded to a draft of 15 feet 6 inches, and to a have a meta centric height of 1·5 feet, we find the arms of the righting levers as follows:—

At 15 feet 6 inches draft the displacement is 1,500 tons. The vertical line A A corresponding to this displacement, is drawn; intersecting the curves $M_0 M_0$, $M_{15} M_{15}$, $M_{30} M_{30}$, &c. A parallel line C C (Fig. 36) is drawn, representing the centre line of ship. The point M_0 is marked off on CC at the same height as the curve $M_0 M_0$ cuts A A. This is the height of the meta centre for this displacement.

The point G is marked off 1·5 feet below M_0, the meta centric height being 1·5 feet. Then G is the centre of gravity of the ship.

The point M_{15} on C C is marked off at the same height as the curve M_{15} M_{15} cuts A A

„ M_{30} „ „ „ $M_{30} M_{30}$ „

„ M_{45} „ „ „ $M_{45} M_{45}$, &c. „

To measure the length of the arm of righting moment at any angle (say 30°) :—Through M_{30} (Fig. 36), draw the line $M_{30} Z_{30}$, inclined at 30° to the centre line. This line will represent the line of action of buoyancy when the ship is inclined 30° from the vertical. The line $G Z_{30}$ drawn through G at right angles to $G_{30} Z_{30}$, therefore, represents the arm of the righting moment. This measures about ·63 of a foot, so that the righting moment at that angle would be

$$= \text{the weight of ship} \times \text{arm of lever} = 1{,}500 \text{ tons} \times \cdot 63 \text{ feet} = 945 \text{ foot tons.}$$

The curve of stability (Fig. 37, Plate IV.) for this ship would be made by laying off the lengths $G Z_{15}$, $G Z_{30}$, $G Z_{45}$, &c., as found from Fig. 36. And the righting moment at any angle is the weight of the ship, multiplied by the height of the curve at that angle.

In Fig. 36 G is below M for all angles up to 90° of heel, and, therefore, the righting moment is positive for all these angles.

The position of the centre of gravity may be anywhere above or below M in the centre line of ship, or on either side of it. The construction for finding the arms of the righting moments will be the same.

Thus, if a diagram similar to Fig. 35 was made by the designer of each ship and supplied to the captain, he would be able to determine the stability of his ship under any condition of loading on ascertaining, by an inclining experiment or by a "stability" gauge (see page 11), the corresponding meta centric height.

The following chapter, which gives a new method of obtaining the diagram (Fig. 35) need not be read by those who only wish to understand the principles involved in calculations of stability.

CHAPTER II.

CENTRES OF BUOYANCY.

To find the centres of buoyancy of ships in inclined positions, by the methods hitherto in use, involves elaborate and tedious calculations of areas, volumes, moments, &c. This labour may be greatly reduced by the use of Amsler's Integrator, which calculates mechanically the areas, moments, &c., but even with the data so found there remains a formidable calculation to deduce from them the required centres of buoyancy.

The usual method is as follows:—The centre of buoyancy, corresponding to a given draft, is found for the ship in her upright position. To find it for any given inclination the position of a water line which gives the same displacement is found.

Next: The volume and moment of the wedge which has been lifted out of the water, and that which is forced into the water by the inclination of the ship, are found.

Next: The alteration in the position of the centre of buoyancy, due to the moments of the immerged and emerged wedges, is calculated on the principles explained for finding the effect of a shifting weight. (See page 9.)

Finally: The position of the centre of buoyancy with ship upright being known, and the alteration in its position being thus found, its position is determined for that angle of inclination, and at that given displacement.

The graphic method to be here given is as follows:—The length of the ship is divided into a number of equal intervals, at the centre of each a cross section is made.

The ship, represented by these cross sections, is placed at any angle of inclination, and is divided into a series of horizontal layers. The area and centre of gravity at the centre of each of these layers is then found.

Then commencing from the lowest layer, and adding up the effect of each successive layer, a centre of buoyancy, corresponding to each displacement, is found for the given angle of inclination. This process being repeated for each 10° or 15° of inclination, and the meta centre being found for the upright position of ship, a diagram is made out containing all the data (depending on the form of the ship) required for determining stability, at any displacement, and at any angle of inclination.

The method of calculating areas, moments, and moments of inertia, by adding the intersects between a pair of lines, or a line and a curve, is so simple and easy in practice that it may be used by any draughtsman, even though he should not be able to follow the mathematical argument given to establish the correctness of the processes. In such a case I would suggest that the sample calculation be carefully studied, when it will be seen that processes which look complicated when expressed algebraically are exceedingly simple in practice. Thus: any point in the curve whose equation is $Y = \frac{x^2}{a\,b}$ may be found by two strokes of the pencil, and involves no more calculation than the drawing of a circle, which is also a conic section.

AREAS OF PLANE CURVES.

All the constructions to be given here are based on the assumption that a curve such as A B C D, Fig. 15, may be treated as equivalent to the rectangles $(a \times x_1) + (a \times x_2) + (a \times x_3) +$ &c. This assumption is not exact; but the amount of probable error involved may be roughly estimated by comparing the area found by this method with the area of curves which we can calculate.

Thus: suppose a water-line surface were a true ellipse (Fig. 16), and its length was 200 feet, breadth = 32 feet, the area would be by calculation = 5,026 square feet; the area would be by method of average = 5,043 square feet, if 20 ordinates were used; giving an error of $\frac{17}{5026}$, which is less than $3\frac{1}{2}$ per 1,000.

If the water-line surface were of the form shown in Fig. 17, the error would be 0.

The water-lines of actual ships may be assumed to be intermediate between these forms; and, if so, we may assume that, by taking the average of 20 ordinates, we should get an approximation within about 2 per 1,000 of the correct area; and this is probably as near an approximation as the ship is to her drawings—that is. as near an approximation as is required for practical purposes.

On this assumption let us find graphically the area contained between the curves A B and C D, and the parallel lines A D and B C, Fig 21.

Through any point P outside of the curve draw the straight line P Q square to B C and A D, divide A the length of the curve into, say seven, equal parts each $= \frac{a}{7}$

At the centre of each of these intervals draw ordinates, and let the length of the ordinates of the curve A B, measured from P Q, be equal to X_1, X_2, X_3, X_4, X_5, X_6, and X_7, and let the ordinates of the curve C D be x_1, x_2, x_3, x_4, x_5, x_6, and x_7.

$$\text{Then the area of A B Q P} = \frac{a}{7}\left\{X_1 + X_2 + \&c., + X_7\right\}$$

$$\text{And the area of C D P Q} = \frac{a}{7}\left\{x_1 + x_2 + \&c., + x_7\right\}$$

and the area of A B C D, which is equal to the difference of these areas:—

$$= \frac{a}{7}(X_1 + X_2 + \&c., + X_7) - \frac{a}{7}(x_1 + x_2 + \&c., + x_7) \ \ldots\ldots\ldots\ldots\ (1)$$

$$\text{and this} = a\left[\frac{X_1}{7} + \frac{X_2}{7} + \&c., + \frac{X_7}{7} - \left(\frac{x_1}{7} + \frac{x_2}{7} + \&c., + \frac{x_7}{7}\right)\right]$$

Now, through any point O in the line P Q draw O M square to P Q, make O M = 7 inches on any scale, and draw M N = 1″ on the same scale. Also through O and N draw the straight line O N. Through each of the points X_1 X_2, &c., and x_1 x_2, &c, the tops of the co-ordinates of the curves A B and C D, draw the lines $(X'_1\, X''_1)$, $(X'_2\, X''_2)$, $(X'_3\, X''_3)$, &c., and $(x'_1\, x''_1)$, $(x'_2\, x''_2)$, $(x'_3\, x''_3)$, &c., parallel to P Q, and intersecting the straight lines O M and O N.

Now, it is evident that each of the lines, such as $X'_3\, X''_3$, is one-seventh of the corresponding ordinate X_3.

So that, if we add all the lines $(X'_1\, X''_1)$ $(X'_2\, X''_2)$, &c., corresponding to the ordinates of the curve A B, we get the sum of the series— $\frac{X_1}{7} + \frac{X_2}{7} + \&c., + \frac{X_7}{7}$

and if we add the lines $(x_1'\, x''_1)$, $(x'_2\, x''_2)$, $(x'_3\, x''_3)$, &c., corresponding to the ordinates of C D, we get the sum of the series $\left(\frac{x_1}{7} + \frac{x_2}{7} + \&c., + \frac{x_7}{7}\right)$

The difference between these sums is therefore $= \left(\frac{X_1}{7} + \frac{X_2}{7} + \&c., + \frac{X_7}{7}\right) - \left(\frac{x_1}{7} + \frac{x_2}{7} + \&c., + \frac{x_7}{7}\right)$

Let this length = m,

Then (by equation 1) the area of curve A B C D = a × m .... (2)

Now, it is evident that the area of any section of the curve is equal to $\frac{a}{7}(X - x)$ and this is equal to $a\left(\frac{X}{7} - \frac{x}{7}\right)$

Therefore the construction given above will give the area of any curve of any length.

If the curve were $\frac{a}{7}$ long, there will be 1 pair of intersects, $X'_1\ X''_1$ and $x'_1\ x''_1$.

If the curve were $2\frac{a}{7}$ long, there will be 2 pairs of intersects, $X'_1\ X''_1$, $X'_2\ X''_2$, and $x'_1\ x''_1$, $x'_2\ x''_2$.

If the curve were $n\frac{a}{7}$ long, there will be n pairs of intersects, $X'_1\ X''_1$, $X'_2\ X''_2$, &c. $X'_n\ X''_n$, &c.,
and $x'_1\ x''_1$, $x'_2\ x''_2$, and $x'_n\ x''_n$.

Where n may be any number.

It must be also observed that as there must be a pair of lines—one positive and one negative—for each ordinate of the curve, if two or more ordinates are of the same length, the corresponding intersect must be added once for each of these ordinates. Thus, if the ordinates X_6 and X_7 have the same intersect it must be added twice, once for X_6 and once for X_7.

In the above we have supposed the length of the curve to be divided by 7. The same reasoning, of course, applies to any other number; so that if n be any number, and co-ordinates are spaced at intervals of $\frac{a}{n}$, and the lines O M and O N drawn so that at any point the ratio of $\frac{O M}{M N} = \frac{n}{1}$, then the sum of the intersects corresponding to the A B curve, minus the sum of the intersects corresponding to the C D curve, gives the height of a rectangle on base (a), which equals the area of the curve A B C D.

This construction will be used for finding the areas of water-line sections, and reducing them all to rectangles on a base equal to the length of the ship.

Thus, if l = the length of the ship, and if d = the distance between the stations at which sections of the ship are made, then one pair of lines, such as O M and O N, drawn so that $\frac{M N}{O M} = \frac{d}{l}$, may be used for reducing all the curves to equivalent rectangles of length = l.

The straight line P Q may be at any distance from the curve; its position may, therefore, be chosen so as to give intersects of suitable lengths.

CENTRES OF GRAVITY OF CURVED AREAS.

Now, assuming as before that a curve may be treated as equivalent to a series of rectangles described about its ordinates, the centre of gravity of any one of the sections into which the curve is divided will be the centre of that co-ordinate. Thus, G_1 (Fig: 21), the centre of gravity of first section, will be half-way between the points X_1 and x_1, and therefore its distance from the line P Q is $\frac{X_1 + x_1}{2}$

Now the moment of this section of the curve about the line P Q is equal to the distance of the centre of gravity multiplied by the area of section. The area $= \left(\frac{X_1 - x_1}{2}\right) \times \frac{a}{7}$: therefore the

$$\text{moment of this section} = \frac{X_1 + x_1}{2} \times (X_1 - x_1)\frac{a}{7} = \frac{a}{2 \times 7}(X_1^2 - x_1^2).$$

The moment of each of the sections being determined in the same way.

The total moment of all the sections $= \dfrac{a}{2 \times 7}\left\{(X_1^2 + X_2^2 + \&c.) - (x_1^2 + x_2^2 + \&c.)\right\}$

But the total moment of all the sections is the moment of the whole curve, and this is equal to the area of curve multiplied by the distance of the centre of gravity of curve from P Q.

Let $x_g =$ this distance of the centre of gravity of curve from P Q.

The area of the curve being a × m (see page 14, equation 2) the moment of the curve $= x_g \times a \times m$, and this is equal to the sum of the moments of all the parts of the curve. That is:

$$x_g \times a \times m = \frac{a}{2 \times 7}\left\{ (X_1^2 + X_2^2 + \&c.) - (x_1^2 + x_2^2 + \&c., + x_7^2);\right. \text{ and, therefore,}$$

$$x_g = \frac{1}{2\,m \times 7}\left\{ (X_1^2 + X_2^2 + \&c.) - (x_1^2 + x_2^2 + \&c.)\right\}$$

To find the value of any term in this series, such as $\dfrac{X_1^2}{2\,m \times 7}$ the following construction is used:

The lines O M and O N (Fig. 22) are drawn (as in Fig. 21) so that $\dfrac{M\,N}{O\,M} = \dfrac{1}{7}$

$O\,X'_1$ is made equal to X_1 then, as before, $X'_1\,X''_1 = \dfrac{X_1}{7}$

O L is made equal to 2 m, and the line L L' drawn square to O M.

From X_1'' the horizontal line $X_1\,H_1''$ is drawn, cutting the line L L' in the point H_1.

So that $L\,H_1 = X'_1\,X''_1 = \dfrac{X_1}{7}$

Through H_1 a line is drawn to O, cutting $X'_1\,X''_1$ in the point Y_1.

Then as the triangle $O\,X'_1\,Y_1$ is similar to the triangle $O\,L\,H_1$:—

$$\frac{X'_1\,Y_1}{O\,X'_1} = \frac{L\,H_1}{O\,L} = \frac{X'_1\,X''_1}{O\,L} = \frac{\frac{X_1}{7}}{2m}\;\frac{X_1}{7 \times 2m} = \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\text{Let } X_1\,Y_1 = Y_1 \text{ then } \frac{X_1\,Y_1}{O_1\,X_1} = \frac{Y_1}{X_1}$$

$$\text{Also } \frac{X_1\,Y_1}{O\,X_1} = \frac{X_1}{7 + 2m} \quad \text{See} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$\text{Therefore } \frac{Y_1}{X_1} = \frac{X_1}{7 \times 2m}$$

$$\text{And } Y_1 = \frac{X_1^2}{7 \times 2m}$$

In the same way, for the distance $O\,X_2$, a height $X_2\,Y_2$ can be determined such that $X_2\,Y_2 = \dfrac{X_2^2}{7 \times 2m}$ or $Y_2 = \dfrac{X_2^2}{7 \times 2m}$

Repeating this process for a series of points X_1, X_2, X_3, &c., it will be found that all the corresponding points Y_1, Y_2, Y_3, &c., lie in a fair curve O Y (this curve being the parabola whose equation is $Y = \dfrac{X^2}{2m \times 7}$)

It will be seen that any point in this curve may be found by two strokes of the pencil, one of which gives the point H the other Y, corresponding to any point X in O M.

If, now, by the above construction, we draw the curve O Y on to Fig. 21, we get Fig. 23, where the points (X_1), (X_2), (X_3), &c., and (x_1), (x_2), (x_3), &c., correspond to the ordinates of A B C D (Fig. 21).

And where the length of the line $X_1{}'Y_1 = \dfrac{X_1{}^2}{7 \times 2m}$

And where the length of the line $X_2{}'Y_2 = \dfrac{X_2{}^2}{7 \times 2m}$

&c., &c. &c., &c.

If we add together all the lines $(X_1{}' Y_1)$, $(X_2{}^1 Y_2)$, $(X^3 Y_3)$, &c.,

" " " " $(x'_1 y_1)$, $(x'_2 y_2)$, $(x'_3 y_3)$, &c.

The difference between these sums

$$= \frac{1}{7 \times 2m} (X_1{}^2 + X_2{}^2 + \&c.) - (x_1{}^2 + x_2{}^2 + \&c.\ x_7{}^2) = x_g \text{ by equation (3)}$$

= the distance of the centre of gravity of the curve A B C D from the line P Q (Fig. 21).

The above construction would be the most convenient if only one curve had to be analyzed; but where we have to find the centres of gravity of a great number of curves, each having different values of m, it requires a different curve O Y to be drawn for each value of m. In finding the centres of gravity of a series of water-lines, we use a slight modification of the above process, so as to make one curve O Y answer for any value of m.

Thus, instead of making O L (Fig. 22) = 2m, it may be any length. Let it = l.

Now, if X is any point in the line O M and X Y is the height of the curve at this point, then, as before, $X\,Y = \dfrac{O\,X_2}{7 \times O\,L} = \dfrac{O\,X^2}{7\,l}$

And the sum of the lines $(X'_1 Y_1)$, $(X'_2 Y_2)$, $(X'_3 Y_3)$, &c.,

Minus the sum of the lines $(x'_1 y_1)$, $(x'_2 y_2)$, $(x'_3 y_3)$, &c.,

$$= \frac{1}{7l} \left\{ (X_1{}^2 + X_2{}^2 + X_3{}^2 + \&c.) - (x_1{}^2 + x_2{}^2 + x_3{}^2 + \&c.) \right\}$$

Let this = x_l

$$\text{Then } \frac{x_l}{x_g} = \frac{\dfrac{1}{7l} (X_1{}^2 + X_2 + \&c.) - (x_1{}^2 + x_2{}^2 + x_3{}^2 + \&c.)}{\dfrac{1}{7 \times 2\,m} (X_1{}^2 + X_2{}^2 + \&c.) - (x_1{}^2 + x_2{}^2 + x_3{}^2 + \&c.)} = \frac{2m}{l}$$

This is the construction that will be used for finding the centre of gravity of water-line sections.

That is, a curve O Y is drawn, so that at any point in that curve $y = \dfrac{x^2}{n\,.\,l}$; then, adding together all

the lines corresponding to the ordinates of one side of the ship, and all those corresponding to the other side, the difference between them is a length which bears the same proportion to the distance of the centre of gravity as the length l used in the construction of the curve bears to 2 m, twice the mean width of the curve.

After having, by the above processes, determined the areas and centres of gravity, corresponding to a series of water-line sections, we adopt another method of determining the displacement and centre of buoyancy corresponding to any draft of water. For obtaining the displacement we proceed on the same principle of estimating the area of each section of a curve as equal to the rectangle formed of the ordinate at the centre of that section, multiplied by the space between the ordinates.

Thus, if the ship is divided into horizontal layers, each 2 feet thick, we assume:—

That the area of the lowest layer is equal to a rectangle 2 ft. thick × the width of the 1 ft. water-line.

The area of the second layer is equal to a rectangle 2 ft. thick × the width of the 3 ft. water-line.

The area of the third layer is equal to a rectangle 2 ft. thick × the width of the 5 ft. water-line, &c.

Thus, we get the displacement at 2 feet, 4 feet, 6 feet, &c., draft of water by adding up the width due to the 1 foot, 3 feet, 5 feet, &c., water-line sections, and multiplying by 2 feet their common thickness.

To add up the actual mean widths of water-lines would give an inconveniently large displacement scale, we, therefore, reduce all these widths in such a proportion as to give the displacement in some convenient scale of tons.

We give a general and particular case, side by side, of finding this proportion, thus:—

Let L = length of ship ... ... = 200 feet.

Let T = thickness of horizontal layers by which displacement is estimated = 2 feet.

Let W = the width of a rectangular mass of water of length = L, and of thickness = T, which would displace 100 tons = 3,500 feet.

That is $W = \dfrac{3{,}500}{\text{L T}}$ or ... ... $\dfrac{3{,}500}{200 \times 2} = 8\frac{3}{4}$ feet.

This is to say, that 100 tons of water are displaced by each W ($8\frac{3}{4}$ feet) in the width of each layer. To reduce this to any convenient scale, such as (n) inches = 100 tons = $\frac{3}{4}$ inches, we draw a pair of lines, C D and D E, Fig. 30, such that $\dfrac{\text{C D}}{\text{D E}} = \dfrac{W \text{ feet}}{\text{(n) inches}} = \dfrac{8\frac{3}{4} \text{ feet (in scale of drawing).}}{\frac{3}{4} \text{ inches (actual measurement.)}}$
This pair of lines will give the displacement (in desired scale) corresponding to any width of water-line section.

In estimating the centres of buoyancy, we assume that the centre of gravity of any horizontal layer is the same as the centre of gravity of the water-line section in the centre of this layer.

Thus, if the ship is divided into layers 2 feet thick, we assume:—

That the centre of gravity of the lowest layer is the centre of gravity of the 1 ft. water-line section.

The centre of gravity of the second layer is the centre of gravity of the 3 ft. water-line section.

The centre of gravity of the third layer is the centre of gravity of the 5 ft. water-line section.

The centre of gravity of the fourth layer is the centre of gravity of the 7 ft. water-line section, &c.

To find the centre of buoyancy at a draught of 4 feet, we find the centre of gravity of two weights, one equal to the displacement of the 1st layer at the 1 foot centre of gravity; the other equal to the displacement of 2nd layer at the 3 feet centre of gravity. This point, the common centre of gravity of the 1st and 2nd layers, is the centre of buoyancy, when these two layers are immersed, that is at 4 feet draught. We next find the centre of gravity on the assumption that one weight, equal to the 4 feet displacement, is concentrated at the 4 feet centre of buoyancy; and another weight, equal to the displacement of the 3rd layer, concentrated at the 5 feet centre of gravity. This gives the centre

of gravity common to the lowest 3 layers, that is the centre of buoyancy at 6 feet draught. Proceeding in the same way, we get in succession the centres of buoyancy corresponding to 8 feet, 10 feet, 12 feet, &c., draughts of water.

The construction used for this purpose is as follows:—If G_a (Fig. 19) is the centre of gravity of a weight $= W_a$, and G_b is the centre of gravity of a weight $= W_b$, these two weights will have a common centre of gravity, G_c, which will be on the straight line, $G_a\,G_b$, and divide it so that

$$\frac{G_c\,G_a}{G_c\,G_b} = \frac{W_b}{W_a}$$

Through G_a and G_b draw the horizontal lines (a G_a)and (b G_b), meeting any vertical line, such as (a b). Make (b d) equal to the weight of W_a on any scale, and (d e) equal W_b, on the same scale. Through the points (a) and (e) draw the straight line (a e), and through (d) draw a parallel straight line (d c), cutting the line (a b) in the point (c); through (c) draw the horizontal line (c G_c), cutting $G_a\,G_b$ in the point G_c, which will be the centre of gravity of the two weights,

$$\text{Because } \frac{G_c\,G_a}{G_c\,G_b} = \frac{c\,a}{c\,b} = \frac{d\,e}{d\,b} = \frac{W_b}{W_a}$$

In the above description, and in the drawing (Fig. 31), the ship is supposed to be divided into horizontal layers, each 2 feet thick; this is done to avoid confusion in the drawing and simplify the description, but in actual practice it is advisable in determining the centres of buoyancy to take the sections much closer than this, especially at and near the lowest part where the displacement varies suddenly.

The assumption that the centre of gravity of any horizontal layer is at the centre of its thickness, is not accurate at any layer, and at the lowest layers it is palpably incorrect, yet this method gives very close approximations, as may be seen by applying it to a ship of triangular section. Thus, if Fig. 18 were the section of a ship, the following table gives the centre of buoyancy by this method of calculation, compared with the true centre of buoyancy.

Draught.	Height of Centre of Buoyancy by Average Method.	Correct Height of Centre of Buoyancy.	Error.
1 foot	6 inches	8 inches	2 inches
2 feet	15 inches	16 inches	1 inch
3 feet	$23\frac{1}{3}$ inches	24 inches	$\frac{2}{3}$ inch
4 feet	$31\frac{1}{2}$ inches	32 inches	$\frac{1}{2}$ inch
5 feet	$39\frac{3}{5}$ inches	40 inches	$\frac{2}{5}$ inch
6 feet	$47\frac{2}{3}$ inches	48 inches	$\frac{1}{3}$ inch
n feet	—	78n inches	$\frac{2}{n}$ inch

From this table it will be seen that, if a ship were of the form of a wedge, by the time we reach launching draught the error is reduced to a small fraction of an inch. If a ship were rectangular instead of wedged shaped, the error would be = O, and as the form of ships are intermediate between a wedge and a rectangle, it is probable that the error involved in this process is intermediate between O and the error in the above table.

By means of the constructions given above we can determine for any angle of inclination, and for any displacement, the corresponding centre of buoyancy.

We will now give a construction for determining the "meta centre" for the upright position of a ship corresponding to any displacement.

CONSTRUCTION FOR FINDING THE META CENTRE.

Fig. 20 is a section of a ship of unit length and of uniform section.

X O X is the water-line when the ship is upright.

X_1 O X_1 is the water-line when the ship is inclined to a small angle $= a^\circ$

B is the centre of buoyancy when the ship is upright.

B_1 is the centre of buoyancy when the ship is inclined a°.

B_1 M is the vertical line through B meeting centre line of ship in the point M, which is the meta centre when the angle (a) is indefinitely small.

Let x = O X = the $\frac{1}{2}$ breadth of ship at water-line.

If the ship's side were at right angles to the water-line, X X_1 would be a straight line at right angles to O X, and would be equal to (x tangent a). Now, if the angle (a) is very small, the error of reckoning X X_1 as equal to (x tangent a) will be very small; if we take X X_1 as equal to (x tangent a), then the area of triangle X O X_1 $= \dfrac{\text{O X} \times \text{X X}_1}{2} = \dfrac{x \times x \text{ tangent a}}{2} = \dfrac{x^2 \text{ tangent a}}{2}$;

The distance of G, the centre of gravity of this triangle from O is $\frac{2}{3}\,x$, and the moment of this triangle about the centre line is its area × the distance of centre of gravity, that is—

$$\text{Moment of triangle} = \frac{x^2 \text{ tangent a}}{2} \times \tfrac{2}{3}\,x = \tfrac{1}{3}\,x^3 \text{ tangent a}.$$

The inclination of the ship to the angle (a) increases the displacement on one side by an amount $= \dfrac{x^2 \text{ tangent a}}{2}$

And decreases the displacement on the other side by an amount $= \dfrac{x^2 \text{ tangent a}}{2}$

And thus the moment about centre line is increased by ($\frac{1}{3}\,x^3$ tangent a) on one side.

And the moment about centre line is decreased by ($\frac{1}{3}\,x^3$ tangent a) on the other side.

The moment on one side is therefore greater by ($\frac{2}{3}\,x^3$ tangent a) than the moment on the other side.

The centre of buoyancy corresponding to this inclination will be shifted to a point such that, if the whole displacement were concentrated at that point, the same moment about the centre line would be produced. If B_1 is this point, and if d = the displacement, then d × B B_1 = $\frac{2}{3}\,x^3$ tangent a; but, B B_1 = (M B_1 tangent a) as B_1 M is inclined at (a°) from the centre line.

$$\text{Therefore, d} \times \text{B B}_1 = \text{d} \times \text{M B}_1 \text{ tangent a} = \tfrac{2}{3}\,x^3 \text{ tangent a}.$$

$$\text{Therefore, M B}_1 = \frac{\frac{2}{3}\,x^3 \text{ tangent a}}{\text{d} \times \text{tangent a}} = \tfrac{2}{3}\,\frac{x^3}{\text{d}}.$$

When the angle (a) is very small. M B is very nearly equal to M B_1; or, assuming them to be equal, we have M B $= \frac{2}{3}\,\dfrac{x^3}{\text{d}}$; that is, the height of the meta centre above the centre of buoyancy $= \frac{2}{3}\,\dfrac{x^3}{\text{d}}$

When the water-line section is a curve, as in ships, we may divide this curve into a series of rectangles by ordinates, spaced at intervals = b. Let the ordinates at centres of these intervals be x_1, x_2, x_3, &c., x_n. Then, as the alteration in moment produced by inclining the ship to angle (a)

Is equal to $\frac{2}{3}$ x^3 tangent a, where there is one section of unit length.

This would be $\frac{2}{3}$ b x^3 tangent a, where there is one section of length = b;

And this would be $\frac{2}{3}$ b tangent a ($x_1{}^3 + x_2{}^3 + x_3{}^3 +$ &c. $x_n{}^3$) where there is a series of such sections.

Also if B is the centre of buoyancy of the ship (as a whole) when upright,

And B_1 „ „ „ „ when inclined,

And if D is the displacement of the ship,

Then, as with a single section (M B_1 tangent a) × D = moment about centre line.

$$= \tfrac{2}{3}\text{ b tangent a } (x_1{}^3 + x_2{}^3 + x_3{}^3 \text{ \&c.} + x_n{}^3)$$

$$\text{And, therefore, M B} = \tfrac{2}{3}\text{ b } \frac{(x_1{}^3 + x_2{}^3 + x_3{}^3 \text{ \&c.} + x_n{}^3)}{D}$$

Now, $\frac{2}{3}$b ($x_1{}^3 + x_2{}^3 + x_3{}^3 +$ &c.) is the moment of inertia of the water-line section about the centre line of ship; therefore,

$$\text{The height of meta centre above centre of buoyancy} = \frac{\textit{moment of inertia of water-line surface}}{\textit{displacement.}}$$

The equation $\text{B M} = \dfrac{\frac{2}{3}\text{b } (x_1{}^3 + x_2{}^3 + x_3{}^3 + \text{\&c.})}{D}$ is solved graphically by a process very similar to that given for finding the centre of gravity of a water-line section. Thus: a curve is drawn whose equation is $z = \dfrac{x_3}{\text{n c d}}$ where n is any number, and c and d are any lengths.

Then the intersects (corresponding to the ordinates x_1 x_2 x_3, &c.) are added up, and give a length which is equal to $\dfrac{1}{\text{n c d}}$ ($x_1{}^3 + x_2{}^3 + x_3{}^3 +$ &c.).

Let this length = Cn.

$$\text{And let B M} = \frac{\frac{2}{3}\text{b } (x_1{}^3 + x_2{}^3 + x_3{}^3 + \text{\&c.})}{D} = \text{Zn}.$$

$$\text{Then } \frac{\text{Zn}}{\text{Cn}} = \frac{\dfrac{\frac{2}{3}\text{b } (x_1{}^3 + x_2{}^3 + x_3{}^3 + \text{\&c.})}{D}}{\dfrac{1}{\text{n c d}}\,(x_1{}^3 + x_2{}^3 + x_3{}^3 + \text{\&c.})} = \frac{\dfrac{\frac{2}{3}\text{ b}}{D}}{\dfrac{1}{\text{n c d}}} = \frac{\frac{2}{3}\text{ b n c d}}{D}$$

Now n is a number, b is a length (being the distance between the ordinates of water-line), and c, d are arbitrary lengths, so that the quantity $\frac{2}{3}$ n b c d is a volume; it may, therefore, be reduced to the same scale as the displacement of the ship, which is also a volume.

Therefore if D = the length which represents the displacement at any water-line;

And if Z = the length which represents $\frac{2}{3}$ n b c d on the same scale;

$$\text{Then } \frac{\text{Zn}}{\text{Cn}} = \frac{\text{Z}}{\text{D}}$$

To draw the curve whose equation is $z = \dfrac{x^3}{n\,c\,d}$

Where z is the height of the curve at any distance x, and where n is any number, and (c) and (d) may be any lengths.

The lines O M, M N, and O N (Fig. 24) are drawn, so that $\dfrac{MN}{OM} = \dfrac{1}{n}$

The length of O C is made = c.

The length of O D is made = d.

Thus, if n = 7, c = 14 feet, d = 18 feet, and the scale of drawing was made $\frac{1}{4}''$ = 1 foot;

Then $\dfrac{MN}{OM} = \dfrac{1}{7}$

O C is measured off = 14 feet to scale of drawing.

O D is measured off = 18 feet to scale of drawing.

Through C and D, verticals C C′ and D D′ are drawn.

P is any point in the line O M.

P Q is drawn square to O M.

From Q, where the vertical through P cuts O N, square on to the line C C′, getting the point H.

Through H draw the line O H, cutting P Q in the point Y.

$$\text{Then } PY = \frac{OP^2}{n \times c} \quad \text{(See Fig. 22.)}$$

From the point Y thus found, square on to the line D D, getting the point K, and through K draw the line O K, cutting P Q in the point Z.

$$\text{Then, as in Fig. 22, } DK = PY = \frac{OP^2}{n \times c}$$

$$\text{And, by similar triangles, } \frac{PZ}{OP} = \frac{DK}{OD}$$

$$= \frac{\;\dfrac{OP^2}{n \times c}\;}{OD} = \frac{\;\dfrac{OP^2}{n\,c}\;}{d} = \frac{OP^2}{n\,c\,d}$$

$$\text{Therefore } PZ = \frac{OP^3}{n\,c\,d}$$

$$\text{Let } OP = x, \text{ and } PZ = z;\ \text{then } z = \frac{x^3}{n\,c\,d}$$

By a repetition of this process, a series of points Z_1, Z_2, Z_3, &c., corresponding to a series of points P_1, P_2, &c., in the line O M, is determined, and the curve O Z, drawn through these points, is a curve whose equation is $z = \dfrac{x^3}{n\,c\,d}$

(SAMPLE CALCULATION.)

We will now give an example of the application of the above processes.

The first thing to be decided is, What parts of the ship are to be considered as adding to its stability? This being determined on practical considerations of strength, &c. Sections, on as large a scale as is practicable, are drawn to include all parts of the ship which are to be reckoned as structurally parts of the hull of the ship, and to exclude all others.

In the example here given the top gallant, forecastle, the bridge, and the poop, are reckoned as assisting in the stability, while hatches, skylights, companions, &c., are ommitted.

The ship is 200 feet long, and divided into segments 10 feet long. At the centre of each of these segments a section of the ship is made, and these are arranged together as in Fig. 28.

No. 1 section being taken at 5 feet from forward perpendiculer;
No. 2 section being taken at 15 feet from " "
No. 3 section being taken at 25 feet from " "
&c. &c.

To find the centres of buoyancy corresponding to any angle of heel, the body plan, Fig. 28, is placed at that angle from the vertical, and a set of horizontal water-lines are drawn at O, 1 foot, 3 feet, 5 feet, &c., draught, commencing at the lowest point of the ship at this inclination.

Below this (Fig. 31) is another set of water-lines for every foot of draught. A A is a vertical line, common to figures 28, 29, and 31; this may be drawn at any distance from the ship.

Fig. 29. O M is a horizontal line, and O N is inclined to it; so that $\frac{\text{M N}}{\text{O M}} = \frac{\text{Pitch of the ordinates}}{\text{Length of ship}} = \frac{10'}{200'} = \frac{1}{20}$

The curve O Y is a parabola constructed as shown in Fig. 22; the arbitrary length (O L in Fig. 22) being taken at 20 feet.

Fig. 30. The line C D is a horizontal line.

The line C E is inclined to it, so that $\frac{\text{D E}}{\text{C D}} = \frac{\frac{3}{4}\text{ inch actual measurement}}{8\frac{3}{4}\text{ ft. (on scale of }\frac{1}{4}'' = 1\text{ ft.)}}$ (See page 18.)

C b is made = to $\frac{1}{2}$ the length used in the construction of curve O Y, that is $= \frac{20'}{2} = 10$ ft.

The pair of lines O M and O N (Fig. 29), serve to find the mean width of any water-line.

The lines C D and C E (Fig. 30), reduce the mean width to displacement for all water-lines.

The curve O Y (Fig. 29), gives for any water-line a length proportioned to the distance of the centre of gravity from A A.

The actual length being found from the lines C D and D E.

To find the mean width of the 11 ft. water-line at this inclination of the ship:—

From each of the points x_1, x_2, x_3, &c., and X_1, X_2, X_3, &c. (Fig. 28), where the 11 ft. water-line cuts the sections of the ship, square on to O M and O N (Fig. 29), getting the lines $(x'_1\, x''_1)$, $(x'_2\, x''_2)$, &c., and $(X'_1\, X''_1)$, $(X'_2\, X''_2)$, &c.

Add up the intersects $(X'_1\, X''_1)$, $(X'_2\, X''_2)$, $(X'_3\, X''_3)$, &c., corresponding to the far side of ship.

And add up the intersects $(x'_1\, x''_1)$, $(x'_2\, x''_2)$, $(x'_3\, x''_3)$, &c., corresponding to the near side of ship.

The difference between these sums is the mean width of the 11 ft. water-line.

Mark off on the line C D (Fig. 30) from the point C, a distance (C b_{11}) equal to this width, and through b_{11} draw a vertical. Then d_{11}, the length of this vertical, intersected between the lines C D and C E, is the displacement of a horizontal layer of 2 ft. thick and of width = b_{11}. This is assumed to be the displacement of the layer contained between the 10 ft. and the 12 ft. water-lines.

To find the centre of gravity of this water-line:—

Add up the lines $(X'_1\,Y_1)$, $(X'_2\,Y_2)$, $(X'_3\,Y_3)$, &c., intersected between the line O M and the curve O Y.
And add up the lines $(x'_1\,y_1)$, $(x'_2\,y_2)$, $(x'_3\,y_3)$, &c., „ „ „ „ „

The difference between these sums : the distance of centre of gravity from line A A.
:: the mean breadth of water line : the constant used in constructing the curve O Y.

Mark off, on the line C D (Fig. 30), a length C a_{11} = the difference between these sums.

Draw a straight line through the points b_{11} and b.

And through the point a_{11} draw a straight line (parallel to the line b_{11} b), cutting C E in g_{11}.

Then the length of the line C g_{11} is equal to the distance of the centre of gravity of the 11 ft. water-line from the line A A.

Mark off this distance from the line A A on the 11 ft. water-line (Fig. 31), getting the point G_{11}, which is assumed as the centre of gravity of the layer contained between the 10 ft. and the 12 ft. water-line.

By a repetition of the above process (d_1), (d_3), (d_5), &c., the displacement for each layer } are found.
And (G_1), (G_3), $(G)_5$, &c., the centre of gravity „ „ }

To construct the displacement scale for this inclination of the ship:—

It is assumed that d_1 = the displacement of the layer between the 0 and the 2 ft. water-line.
„ d_3 = „ „ „ 2 ft. „ 4 ft. „
„ d_5 = „ „ „ 4 ft. „ 6 ft. „

Therefore the displacement at 2 ft. draught = d_1
„ „ 4 ft. draught = $d_1 + d_3$
„ „ 6 ft. draught = $d_1 + d_3 + d_5$
„ „ 8 ft. draught = $d_1 + d_3 + d_5 + d_7$
&c. = &c. &c.

On the 2 ft. water-line (Fig. 31) mark off a distance from A A = d_1 getting the point D_2.
On the 4 ft. „ „ „ „ = $d_1 + d_3$ „ D_4.
On the 6 ft. „ „ „ „ = $d_1 + d_3 + d_5$ „ D_6.
&c. = &c. &c.

A curve swept through the points D_2, D_4, D_6, &c., is, approximately, the displacement scale for this inclination of the ship.

To find the centres of buoyancy at the several draughts of water:—

It is assumed that G_1 is the centre of gravity of the first layer; it will, therefore, be the centre of buoyancy of that layer. That is, B_2 coincides with G_1.

To find the centre of buoyancy at 4 ft. draught, or the point B_4, we find the centre of gravity of two weights, one equal to d_1 at G_1, the other equal to d_3 at G_3.

To find B_6 we find the centre of gravity of two weights, one equal to D_4 at B_4, the other $= d_5$ at G_5.
To find B_8 „ „ „ D_6 at B_6, „ $= d_7$ at G_7.

The above is done by a repetition of the process explained in Fig. 19.

Thus: as G_1 is assumed to be the centre of buoyancy at 2 ft., B_2 the centre of buoyancy at 2 ft., coincides with G_1.

To find the centre of buoyancy at 4 ft. draught:—

Through the point where the 1 ft. water-line cuts A A and the point C_3 in the 3 ft. water-line draw a straight line, and through the point C_2 in the 3 feet water-line draw the line $C_2 B_4$ (parallel to $B_2 C_3$), and cutting A A in B'_4.

Then B'_4 is the height of the centre of buoyancy at 4 ft. draught.

From this point (B'_4) square on to the line joining G_3 and B_2, getting the point B_4, which is assumed to be the centre of buoyancy at 4 feet draught.

In the same way, through the point C_4 in the 5 feet water-line—

Draw, parallel to $B'_4 C_5$, the line $C_4 B'_6$, cutting A A in B_6, which gives the height of B_6, and square from B'_6 on to the line joining B_4 and G_5, getting the point B_6, the centre of buoyancy at 6 ft. draught.

A repetition of this process gives the points (B_8), (B_{10}), &c., the centres of buoyancy at 8 ft., 10 ft., 12 ft., &c., draughts of water.

Now, the arm of righting lever, when a ship is inclined, is the distance between the vertical through the centre of buoyancy and the vertical through the centre of gravity of the ship.

If, then, we know any point in the vertical through the centre of buoyancy, this point will fix the line of action of buoyancy.

Let us find the point where the vertical through the centre of buoyancy cuts the centre line of ship.

In (Fig. 31) the centre line of ship is drawn to occupy the same position relatively to A A and the base line that the centre line of ship occupies relatively to A A and the base line in Fig. 28, and from each of the points B_4, B_6, B_8, &c., &c., vertical lines are drawn, intersecting the centre line of ship in the points M_4, M_6, M_8, M_{10}, &c., &c.

To complete the analysis of the ship at this angle of inclination, it is now only necessary to draw the curve M'_6, M'_8,——M'_{22}, which gives the height of M corresponding to any displacement.

Through D_6, the point where the 6 feet water-line intersects the curve of displacement, draw the vertical line $M'_6 D_6$; make the height of M'_6 above the base line equal to the height of M_6 above the top of keel.

Also, through D_8, D_{10}, &c., D_{22}, draw verticals, and on them mark off M'_8, M'_{10}, &c., M'_{22}, corresponding to the heights of M_8, M_{10}, &c., M_{22} above the top of keel, and sweep a curve through the points M'_6, M'_8, M'_{10}, &c. This curve gives the height of the point M where the vertical through the centre of buoyancy intersects the centre line of ship corresponding to any displacement at this angle of inclination.

Having now finished with the ship at this angle, Fig. 28 may be turned round to any other angle, and a diagram similar to Fig. 31, corresponding to that angle, may be made; and this process may be repeated till we have a complete series of such diagrams—one for each 10° or 15° of heel.

CONSTRUCTION FOR THE META CENTRE.

The methods given above apply to any angle of heel; but, for the upright position, another construction, that for finding the meta centre, is necessary.

When the ship is upright, all the centres of buoyancy are in the centre line of ship, and the two sides are alike; it is, therefore, only necessary to analyze one side of the ship.

The constructions used for this purpose are shown in Figs. 32 and 33.

Fig. 32.
- The lines O M and O N are drawn so that $\frac{\text{M N}}{\text{O M}} = \frac{2}{20} = \frac{1}{10}$.
- The curve O Z is a cubic parabola, constructed as shown in Fig. 24, where n = 10.
- And the arbitrary length O C, in Fig. 24, has been taken as = $17\frac{1}{2}$ feet.
- „ „ „ O D, „ „ „ = 20 feet.

Fig. 33.
- The line C D is a horizontal line.
- The line C E is inclined to it, so that $\frac{\text{D E}}{\text{C D}} = \frac{\frac{3}{8}'' \text{ actual measurement.}}{8\frac{3}{4} \text{ feet on scale of drawing.}}$ (See page 23.)
- And C Z is made = $\frac{2}{3}$ n b c d = Z, (See page 21.)

In this case n = $\frac{\text{O M}}{\text{M N}}$ = 10.

b = 10 ft., the pitch of the ordinates of the ship's sections.

c = $17\frac{1}{2}$ ft., the arbitrary constant used in the construction of curve O Z.

d = 20 ft. the arbitrary constant used in the construction of curve O Z.

And, therefore, $\frac{2}{3}$ n b c d = $\frac{2}{3}$ 10 × 10′ × $17\frac{1}{2}$′ × 20′ cubic ft.

$$\text{And this} = \frac{\frac{2}{3}\ 10 \times 10 \times 17\tfrac{1}{2} \times 20}{35} \text{ tons.}$$

= 666 tons.

That is, C Z = 666 tons on the same displacement scale as that of the ship—viz., $\frac{3}{8}''$ = 100 tons.

The lines O M and O N give the mean breadth of any water-line section of the upright position.

The lines C D and C E give the displacement corresponding to this mean breadth.

The curve O Z gives a multiple of the height of the meta centre for any water-line.

The lines C D and C E reduce this multiple to the actual height.

Thus, to find the mean width of the 11 feet water-line (the ship being vertical):—

On the line O M (Fig. 32) the points x'_1, x'_2, x'_3, &c., are marked off, at distances from O equal to the ordinates of the half-breadth plan (of the 11 feet water-line in upright position) measured from the centre line of ship.

Through each of the points x'_1, x'_2, x'_3, &c., verticals are drawn, cutting the line O N in the points x''_1, x''_2, x''_3, &c.

The lines $(x'_1\ x''_1)$, $(x'_2\ x''_2)$, $(x'_3\ x''_3)$, &c., intersected between the lines O M and O N, are added together, and give twice the mean breadth of the half-breadth plan of water-line.

That is the mean breadth of the water-line section. Let this = b_{11}.

Then, on the line C D (Fig. 33) the length C b_{11} is marked off, = b_{11}.

And the vertical at this point intersected between O M and O N gives the displacement, = d_{11}, of a layer 2 feet thick of the mean width of the 11 feet water-line, and this is assumed as the displacement between the 10 ft. and the 12 ft. water-lines.

By a repetition of the above (d_1, d_3, d_5, &c.) the displacements of the first, second, third, &c., layers are found, and from these the displacement curve D D is drawn for the upright position of ship in the same way as for inclined positions.

Also, the heights of the centres of buoyancy corresponding to different displacements are found precisely as the heights were found for inclined positions.

The heights of the centres of buoyancy being determined, the curve B B (Fig. 34) is drawn thus:—

On the vertical through D_6 corresponding to the displacement at 6 ft. draught, the point B_6 is marked off at a height = the height of the centre of buoyancy at 6 ft. draught.

And on the vertical through D_8 the point B_8 is marked at a height = B'_8.

" " " D_{10} " B_{10} " " = B'_{10}.

&c. &c. &c.

The curve B B drawn through these points gives the height of the centre of buoyancy corresponding to any displacement.

To find the height of the meta centre corresponding to a draught of 11 feet.

Add together the intersects (Fig. 32) between the lines O M and the curve O Z, getting the length c_{11}.

On the line C D (Fig. 33) lay off the length C c_{11} = to c_{11} thus found.

And " " " C D_{11} = the displacement on displacement scale corresponding to a draught of 11 feet.

Through the points Z and D_{11} draw a straight line, Z D_{11}.

And through the point c_{11} draw a line (parallel to Z D_{11}, cutting C E in the point Z_{11}.

Then the length of C Z_{11} will be equal to the height of the meta centre above the centre of buoyancy at a draught of 11 ft.

Through the point where the 11 ft. water-line cuts displacement curve (Fig. 34) draw the vertical B_{11} M_{11}, and mark off above the point B_{11} the length B_{11} M_{11}, equal to the length C Z_{11}, found as above; then M_{11} will be the meta centre corresponding to a draught of 11 feet.

In the same way the points M_7, M_9, M_{13}, &c., are determined, corresponding to the heights of the meta centre at draughts of 7 feet, 9 feet, 13 feet, &c.

The curve M M, drawn through these points, gives the height of the meta centre corresponding to any displacement.

Having now given the processes for finding the height of the meta centre corresponding to any displacement, and also for finding the height of the intersection of the centre line of ship with the vertical through the centre of buoyancy for any displacement and for any inclination.

We will now give the construction of the diagram (Fig. 35) which combines all these results in one.

The displacement curve D D (Fig. 35) is traced direct from the displacement curve D D (Fig. 34). (N.B.—For convenience of printing, Figs. 34 and 31 have been reduced to one-half the scale of Fig. 34.)

And the curve M_0 M_0 (Fig. 35) is traced direct from the curve M M (Fig. 34), giving the height of the meta centre corresponding to any displacement.

And the curve M_{30} M_{30} (Fig. 35) is traced direct from the curve M M (Fig. 31), giving the height of intersection of the centre line of ship with vertical through centre of buoyancy corresponding to any displacement at an inclination of the ship equal to 30° from the vertical.

In the same way the curves $(M_{15}\ M_{15})$, $(M_{45}\ M_{45})$, &c., are traced directly from diagrams similar to Fig. 31, made for these inclinations of the ship.

Then the diagram (Fig. 35) gives all the data required for determining the stability of the ship at any inclination and for any displacement. (See page 11.)

In working out the above processes, great care and skill is required to get approximations that can be relied on.

The principal source of error is due to inaccuracy of working; but this is common to all processes which have been or can be devised.

The error inherent in the system can be reduced to a quantity too small to be perceived.

Skill is required, especially in the spacing of the ordinates, and allowance for deck erections.

One way of diminishing the error due to this source is, to cut off the two ends of the ship, and any parts of the deck erections which do not terminate at the planes by which the ship is divided into segments (see page 23), and to estimate separately for the ends thus cut off and for the rest of the ship. But it would be impossible to give formulas for treating every separate case.

I have only attempted to explain the principles of the process. In actual practice many variations and qualifications will require to be supplied by the draughtsman. If a clear idea of the principles involved has been conveyed, these details can be supplied by any competent draughtsman. If I have failed to convey these ideas, the addition of further details would only make the subject more complicated.

It will be seen that the whole process given above involves no calculations but what are made with the T square and scale.

It is applicable to drawings made to any scale and with any number of ordinates. To reduce the errors to a minimum, it might be applied to the full size drawing of the ship on the mould loft floor.

A curve such as O Y (Fig. 29) being once drawn, gives the centre of gravity, not only for all the water-lines of one ship, but for any water-line of any ship.

And a curve such as O Z (Fig. 32), in the same way may be used to find the height of the meta centre for any water-line for any ship.

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PLATE 1.

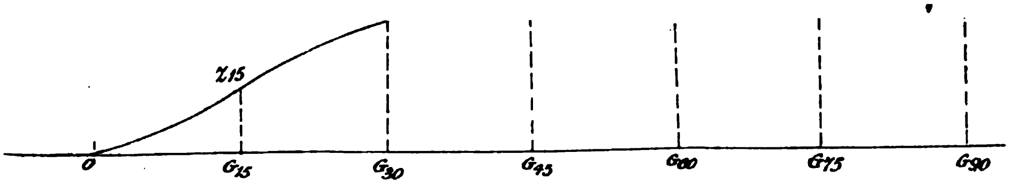

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FIG. 10

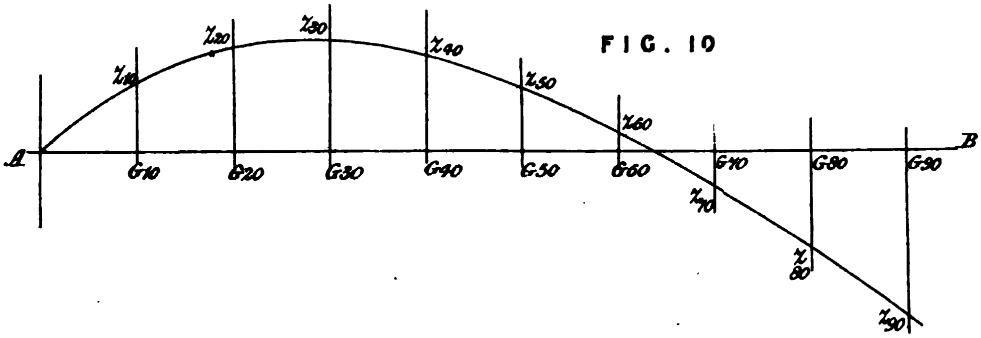


FIG. 11

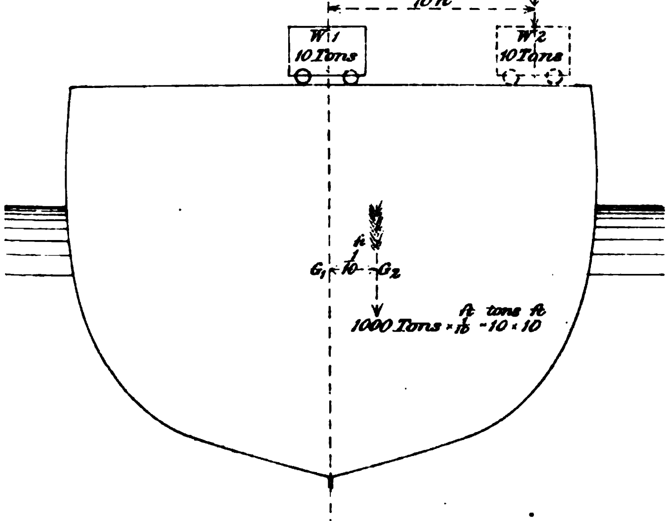


FIG. 12

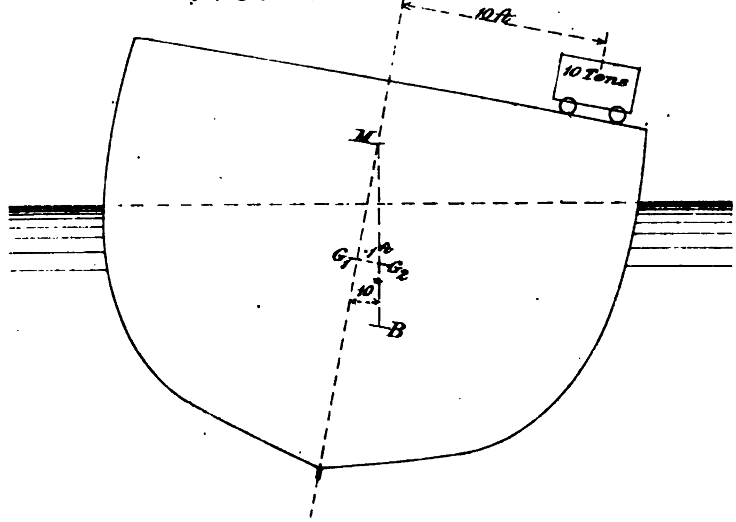


FIG. 13

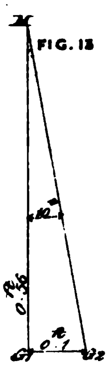


FIG. 14

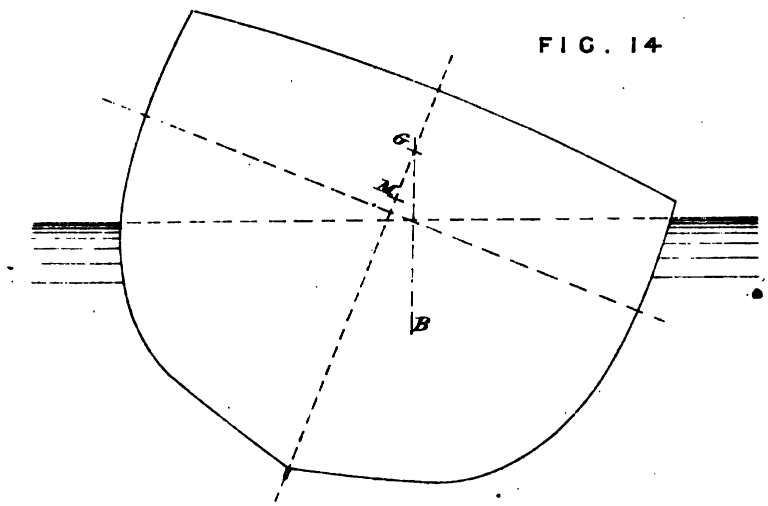
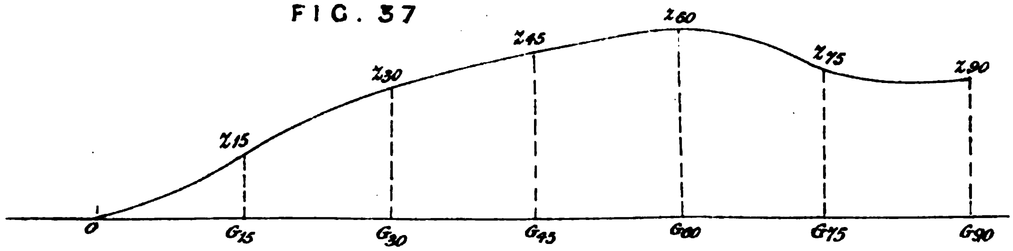
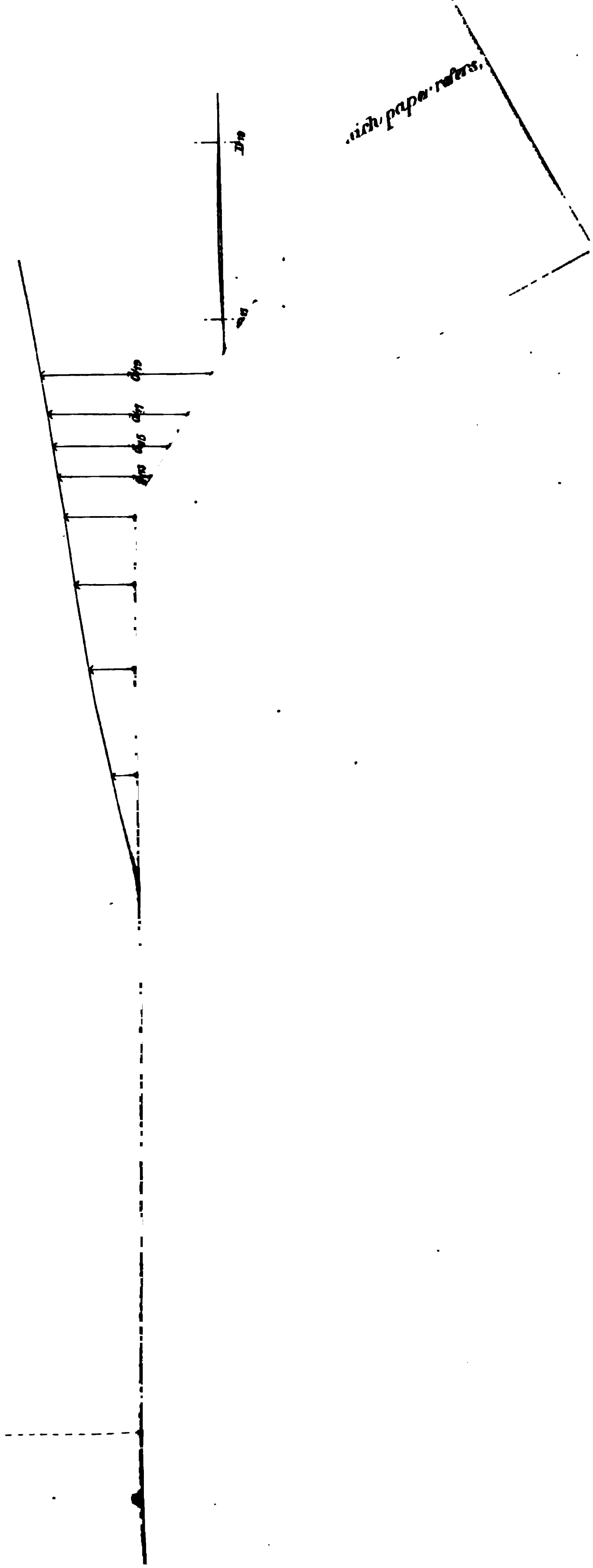
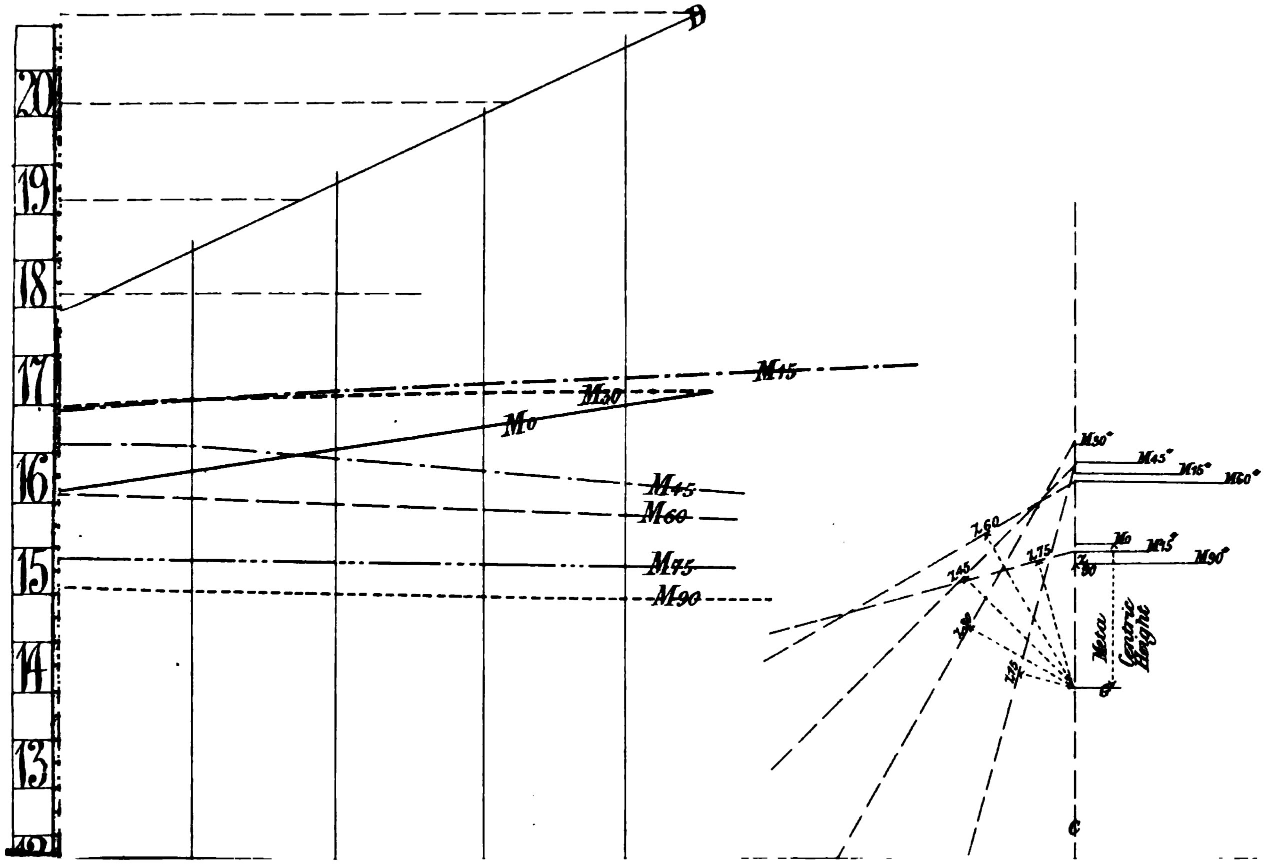


FIG. 37



N.B. Scale reduced to $\frac{1}{2}$ of original drawings to which paper refers
displacement scale - $\frac{3}{8}$" - 100 tons
all other dimensions to a scale - $\frac{1}{4}$" - 1 foot.


paper refers,



20
19
18
17
16
15
14
13
D
M15
M30
M0
M45
M60
M75
M90
M30°
M45°
M15°
M60°
M0
M75°
M90°
Meta
Centric
Height
C